**Hyperparameter optimization results of each machine learning model**

**1. Gaussian Process Regression (GPR)**

Optimization range: “s”: [0.01, 1], “l”: [0.01, 0.5], “sigma”: [0.01, 0.1].

**Table 1.1.**GPR hyperparameters for different orders of POD coefficients of temperature.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “s” | “l” | “sigma” |
|  | 1 | 0.64 | 0.50 | 0.02 |
|  | 2 | 0.52 | 0.50 | 0.01 |
|  | 3 | 0.59 | 0.04 | 0.01 |
|  | 4 | 0.57 | 0.13 | 0.10 |
|  | 5 | 0.43 | 0.06 | 0.10 |
|  | 1 | 0.52 | 0.50 | 0.04 |
|  | 2 | 0.72 | 0.15 | 0.10 |
|  | 3 | 0.41 | 0.04 | 0.10 |
|  | 4 | 0.62 | 0.05 | 0.01 |
|  | 5 | 0.39 | 0.04 | 0.01 |
|  | 1 | 0.63 | 0.05 | 0.10 |
|  | 2 | 0.55 | 0.05 | 0.10 |
|  | 3 | 0.42 | 0.05 | 0.10 |
|  | 4 | 0.56 | 0.06 | 0.10 |
|  | 5 | 0.53 | 0.08 | 0.10 |

**Table 1.2.**GPR hyperparameters for different orders of POD coefficients of pressure.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “s” | “l” | “sigma” |
|  | 1 | 0.50 | 0.05 | 0.10 |
|  | 2 | 0.46 | 0.23 | 0.10 |
|  | 3 | 0.45 | 0.04 | 0.03 |
|  | 4 | 0.52 | 0.05 | 0.01 |
|  | 5 | 0.61 | 0.07 | 0.10 |
|  | 1 | 0.47 | 0.04 | 0.01 |
|  | 2 | 0.57 | 0.05 | 0.10 |
|  | 3 | 0.48 | 0.20 | 0.10 |
|  | 4 | 0.41 | 0.05 | 0.10 |
|  | 5 | 0.43 | 0.07 | 0.05 |
|  | 1 | 0.57 | 0.05 | 0.10 |
|  | 2 | 0.53 | 0.05 | 0.10 |
|  | 3 | 0.44 | 0.07 | 0.10 |
|  | 4 | 0.56 | 0.07 | 0.10 |
|  | 5 | 0.52 | 0.05 | 0.10 |

**Table 1.3.**GPR hyperparameters for different orders of POD coefficients of the x-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “s” | “l” | “sigma” |
|  | 1 | 0.59 | 0.06 | 0.10 |
|  | 2 | 0.46 | 0.05 | 0.10 |
|  | 3 | 0.51 | 0.05 | 0.10 |
|  | 4 | 0.49 | 0.08 | 0.10 |
|  | 5 | 0.46 | 0.05 | 0.10 |
|  | 1 | 0.47 | 0.05 | 0.01 |
|  | 2 | 0.50 | 0.04 | 0.01 |
|  | 3 | 0.53 | 0.08 | 0.10 |
|  | 4 | 0.39 | 0.05 | 0.08 |
|  | 5 | 0.49 | 0.05 | 0.10 |
|  | 1 | 0.59 | 0.05 | 0.10 |
|  | 2 | 0.50 | 0.04 | 0.01 |
|  | 3 | 0.47 | 0.05 | 0.10 |
|  | 4 | 0.46 | 0.05 | 0.10 |
|  | 5 | 0.57 | 0.06 | 0.10 |

**Table 1.4.**GPR hyperparameters for different orders of POD coefficients of the y-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “s” | “l” | “sigma” |
|  | 1 | 0.60 | 0.06 | 0.01 |
|  | 2 | 0.52 | 0.05 | 0.10 |
|  | 3 | 0.45 | 0.04 | 0.03 |
|  | 4 | 0.55 | 0.19 | 0.10 |
|  | 5 | 0.55 | 0.05 | 0.10 |
|  | 1 | 0.53 | 0.05 | 0.01 |
|  | 2 | 0.63 | 0.08 | 0.10 |
|  | 3 | 0.51 | 0.05 | 0.10 |
|  | 4 | 0.42 | 0.05 | 0.10 |
|  | 5 | 0.52 | 0.07 | 0.10 |
|  | 1 | 0.60 | 0.05 | 0.09 |
|  | 2 | 0.50 | 0.04 | 0.01 |
|  | 3 | 0.49 | 0.05 | 0.10 |
|  | 4 | 0.46 | 0.05 | 0.06 |
|  | 5 | 0.42 | 0.08 | 0.09 |

**Table 1.5.**GPR hyperparameters for different orders of POD coefficients of the z-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “s” | “l” | “sigma” |
|  | 1 | 0.59 | 0.06 | 0.10 |
|  | 2 | 0.53 | 0.05 | 0.10 |
|  | 3 | 0.55 | 0.05 | 0.10 |
|  | 4 | 0.48 | 0.09 | 0.07 |
|  | 5 | 0.49 | 0.05 | 0.10 |
|  | 1 | 0.46 | 0.04 | 0.01 |
|  | 2 | 0.45 | 0.07 | 0.10 |
|  | 3 | 0.52 | 0.04 | 0.01 |
|  | 4 | 0.60 | 0.19 | 0.10 |
|  | 5 | 0.49 | 0.05 | 0.10 |
|  | 1 | 0.62 | 0.05 | 0.10 |
|  | 2 | 0.55 | 0.05 | 0.10 |
|  | 3 | 0.43 | 0.05 | 0.10 |
|  | 4 | 0.59 | 0.06 | 0.10 |
|  | 5 | 0.44 | 0.07 | 0.10 |

**2. Support Vector Machine (SVM)**

Optimization range: “C”: [1, 100], “epsilon”: [0.001, 1], “gamma”: [1, 1000].

**Table 2.1.**SVM hyperparameters for different orders of POD coefficients of temperature.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “C” | “epsilon” | “gamma” |
|  | 1 | 61.037 | 0.001 | 65.737 |
|  | 2 | 62.823 | 0.001 | 59.430 |
|  | 3 | 55.124 | 0.186 | 969.615 |
|  | 4 | 61.273 | 0.177 | 65.833 |
|  | 5 | 83.881 | 0.114 | 181.778 |
|  | 1 | 50.184 | 0.001 | 52.763 |
|  | 2 | 15.706 | 0.001 | 326.183 |
|  | 3 | 78.956 | 0.001 | 992.477 |
|  | 4 | 59.087 | 0.149 | 157.126 |
|  | 5 | 11.454 | 0.101 | 990.547 |
|  | 1 | 36.488 | 0.117 | 863.240 |
|  | 2 | 62.489 | 0.392 | 461.236 |
|  | 3 | 53.340 | 0.095 | 386.630 |
|  | 4 | 20.771 | 0.001 | 326.258 |
|  | 5 | 60.503 | 0.151 | 156.334 |

**Table 2.2.**SVM hyperparameters for different orders of POD coefficients of pressure.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “C” | “epsilon” | “gamma” |
|  | 1 | 51.800 | 0.001 | 29.650 |
|  | 2 | 34.770 | 0.001 | 330.400 |
|  | 3 | 89.400 | 0.063 | 983.600 |
|  | 4 | 62.680 | 0.302 | 64.470 |
|  | 5 | 59.980 | 0.175 | 157.100 |
|  | 1 | 55.120 | 0.186 | 969.600 |
|  | 2 | 77.170 | 0.001 | 984.700 |
|  | 3 | 13.790 | 0.018 | 367.600 |
|  | 4 | 44.580 | 0.123 | 495.700 |
|  | 5 | 62.440 | 0.001 | 160.300 |
|  | 1 | 39.480 | 0.272 | 828.900 |
|  | 2 | 66.460 | 0.457 | 471.900 |
|  | 3 | 21.410 | 0.001 | 345.700 |
|  | 4 | 83.580 | 0.136 | 182.200 |
|  | 5 | 15.290 | 0.254 | 366.800 |

**Table 2.3.**SVM hyperparameters for different orders of POD coefficients of the x-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “C” | “epsilon” | “gamma” |
|  | 1 | 20.550 | 0.001 | 705.600 |
|  | 2 | 28.920 | 0.009 | 610.100 |
|  | 3 | 1.000 | 0.031 | 1000.000 |
|  | 4 | 58.590 | 0.245 | 155.500 |
|  | 5 | 21.230 | 0.267 | 73.920 |
|  | 1 | 55.120 | 0.186 | 969.600 |
|  | 2 | 1.000 | 0.100 | 1000.000 |
|  | 3 | 25.320 | 0.001 | 330.600 |
|  | 4 | 61.740 | 0.212 | 66.560 |
|  | 5 | 52.750 | 0.428 | 26.390 |
|  | 1 | 12.870 | 0.338 | 943.000 |
|  | 2 | 39.430 | 0.057 | 984.000 |
|  | 3 | 62.230 | 0.282 | 64.950 |
|  | 4 | 62.720 | 0.001 | 972.500 |
|  | 5 | 40.690 | 0.001 | 518.200 |

**Table 2.4.**SVM hyperparameters for different orders of POD coefficients of the y-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “C” | “epsilon” | “gamma” |
|  | 1 | 62.520 | 0.293 | 64.540 |
|  | 2 | 60.420 | 0.002 | 684.100 |
|  | 3 | 13.840 | 0.181 | 367.900 |
|  | 4 | 52.770 | 0.001 | 381.500 |
|  | 5 | 52.760 | 0.441 | 26.410 |
|  | 1 | 12.870 | 0.338 | 943.000 |
|  | 2 | 31.580 | 0.001 | 685.100 |
|  | 3 | 11.450 | 0.101 | 990.500 |
|  | 4 | 52.900 | 0.203 | 26.420 |
|  | 5 | 44.390 | 0.001 | 275.100 |
|  | 1 | 22.350 | 0.116 | 969.400 |
|  | 2 | 80.140 | 0.435 | 10.590 |
|  | 3 | 52.980 | 0.300 | 26.450 |
|  | 4 | 47.010 | 0.001 | 499.500 |
|  | 5 | 64.740 | 0.001 | 268.700 |

**Table 2.5.**SVM hyperparameters for different orders of POD coefficients of the z-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “C” | “epsilon” | “gamma” |
|  | 1 | 71.100 | 0.022 | 969.900 |
|  | 2 | 19.590 | 0.292 | 985.400 |
|  | 3 | 52.400 | 0.320 | 26.490 |
|  | 4 | 60.650 | 0.043 | 66.310 |
|  | 5 | 33.000 | 0.140 | 971.000 |
|  | 1 | 55.120 | 0.186 | 969.600 |
|  | 2 | 27.100 | 0.001 | 283.800 |
|  | 3 | 100.000 | 0.100 | 1000.000 |
|  | 4 | 23.250 | 0.086 | 290.600 |
|  | 5 | 81.790 | 0.321 | 8.378 |
|  | 1 | 4.424 | 0.337 | 949.100 |
|  | 2 | 100.000 | 0.001 | 1.000 |
|  | 3 | 23.350 | 0.122 | 290.200 |
|  | 4 | 64.280 | 0.001 | 461.300 |
|  | 5 | 50.770 | 0.052 | 279.400 |

**3. Back Propagation Neural Network (BPNN)**

Optimization range: “layers”: [2, 3], “neurons”: [20, 50], “l2”: [10-5, 10-4, 10-3].

**Table 3.1.**BPNN hyperparameters for different orders of POD coefficients of temperature.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “layers” | “neurons” | “l2” |
|  | 1 | 3 | 22 | 10-5 |
|  | 2 | 3 | 26 | 10-5 |
|  | 3 | 3 | 34 | 10-5 |
|  | 4 | 3 | 22 | 10-3 |
|  | 5 | 3 | 44 | 10-5 |
|  | 1 | 3 | 22 | 10-4 |
|  | 2 | 2 | 44 | 10-4 |
|  | 3 | 3 | 46 | 10-4 |
|  | 4 | 3 | 30 | 10-3 |
|  | 5 | 2 | 38 | 10-5 |
|  | 1 | 2 | 50 | 10-5 |
|  | 2 | 3 | 46 | 10-4 |
|  | 3 | 3 | 28 | 10-4 |
|  | 4 | 2 | 30 | 10-5 |
|  | 5 | 3 | 40 | 10-4 |

**Table 3.2.**BPNN hyperparameters for different orders of POD coefficients of pressure.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “layers” | “neurons” | “l2” |
|  | 1 | 3 | 30 | 10-5 |
|  | 2 | 3 | 48 | 10-4 |
|  | 3 | 3 | 46 | 10-5 |
|  | 4 | 3 | 30 | 10-4 |
|  | 5 | 3 | 30 | 10-5 |
|  | 1 | 3 | 46 | 10-5 |
|  | 2 | 2 | 30 | 10-4 |
|  | 3 | 3 | 46 | 10-5 |
|  | 4 | 2 | 38 | 10-4 |
|  | 5 | 3 | 46 | 10-3 |
|  | 1 | 2 | 48 | 10-4 |
|  | 2 | 3 | 30 | 10-4 |
|  | 3 | 2 | 38 | 10-4 |
|  | 4 | 2 | 50 | 10-5 |
|  | 5 | 3 | 30 | 10-5 |

**Table 3.3.**BPNN hyperparameters for different orders of POD coefficients of the x-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “layers” | “neurons” | “l2” |
|  | 1 | 3 | 30 | 10-4 |
|  | 2 | 3 | 46 | 10-5 |
|  | 3 | 3 | 30 | 10-5 |
|  | 4 | 3 | 22 | 10-5 |
|  | 5 | 3 | 30 | 10-4 |
|  | 1 | 2 | 38 | 10-5 |
|  | 2 | 3 | 46 | 10-3 |
|  | 3 | 3 | 30 | 10-5 |
|  | 4 | 2 | 24 | 10-5 |
|  | 5 | 2 | 38 | 10-4 |
|  | 1 | 3 | 44 | 10-5 |
|  | 2 | 3 | 46 | 10-5 |
|  | 3 | 2 | 48 | 10-5 |
|  | 4 | 2 | 38 | 10-4 |
|  | 5 | 3 | 46 | 10-3 |

**Table 3.4.**BPNN hyperparameters for different orders of POD coefficients of the y-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “layers” | “neurons” | “l2” |
|  | 1 | 3 | 30 | 10-5 |
|  | 2 | 3 | 46 | 10-3 |
|  | 3 | 2 | 44 | 10-5 |
|  | 4 | 3 | 22 | 10-5 |
|  | 5 | 2 | 22 | 10-5 |
|  | 1 | 3 | 46 | 10-5 |
|  | 2 | 3 | 30 | 10-3 |
|  | 3 | 3 | 46 | 10-5 |
|  | 4 | 2 | 22 | 10-5 |
|  | 5 | 2 | 38 | 10-5 |
|  | 1 | 2 | 44 | 10-5 |
|  | 2 | 3 | 34 | 10-5 |
|  | 3 | 2 | 38 | 10-5 |
|  | 4 | 2 | 38 | 10-4 |
|  | 5 | 2 | 30 | 10-5 |

**Table 3.5.**BPNN hyperparameters for different orders of POD coefficients of the z-velocity.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Mode order | “layers” | “neurons” | “l2” |
|  | 1 | 2 | 38 | 10-4 |
|  | 2 | 3 | 30 | 10-5 |
|  | 3 | 2 | 22 | 10-5 |
|  | 4 | 3 | 46 | 10-5 |
|  | 5 | 3 | 30 | 10-5 |
|  | 1 | 3 | 46 | 10-3 |
|  | 2 | 3 | 46 | 10-5 |
|  | 3 | 3 | 30 | 10-4 |
|  | 4 | 2 | 30 | 10-4 |
|  | 5 | 2 | 32 | 10-4 |
|  | 1 | 2 | 44 | 10-5 |
|  | 2 | 3 | 44 | 10-4 |
|  | 3 | 3 | 26 | 10-5 |
|  | 4 | 2 | 30 | 10-5 |
|  | 5 | 2 | 38 | 10-3 |

**4. Radial Basis Function Neural Network (RBFNN)**

Optimization range: “l2”: [10-5, 10-4, 10-3], “num\_centers”: [2, 20]. The scale and center of the Gaussian kernel can be obtained by backpropagation, and the specific values are omitted here.

**Table 4.1.**RBFNN hyperparameters for different orders of POD coefficients of temperature.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mode order | “l2” | “num\_centers” |
|  | 1 | 10-5 | 8 |
|  | 2 | 10-5 | 2 |
|  | 3 | 10-5 | 6 |
|  | 4 | 10-5 | 3 |
|  | 5 | 10-4 | 19 |
|  | 1 | 10-5 | 6 |
|  | 2 | 10-3 | 6 |
|  | 3 | 10-4 | 17 |
|  | 4 | 10-3 | 10 |
|  | 5 | 10-5 | 19 |
|  | 1 | 10-5 | 3 |
|  | 2 | 10-4 | 17 |
|  | 3 | 10-5 | 5 |
|  | 4 | 10-5 | 3 |
|  | 5 | 10-3 | 19 |

**Table 4.2.**RBFNN hyperparameters for different orders of POD coefficients of pressure.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mode order | “l2” | “num\_centers” |
|  | 1 | 10-3 | 2 |
|  | 2 | 10-3 | 5 |
|  | 3 | 10-5 | 2 |
|  | 4 | 10-4 | 4 |
|  | 5 | 10-3 | 16 |
|  | 1 | 10-3 | 3 |
|  | 2 | 10-4 | 6 |
|  | 3 | 10-4 | 6 |
|  | 4 | 10-3 | 2 |
|  | 5 | 10-3 | 13 |
|  | 1 | 10-5 | 2 |
|  | 2 | 10-5 | 2 |
|  | 3 | 10-5 | 5 |
|  | 4 | 10-5 | 16 |
|  | 5 | 10-4 | 12 |

**Table 4.3.**RBFNN hyperparameters for different orders of POD coefficients of the x-velocity.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mode order | “l2” | “num\_centers” |
|  | 1 | 10-4 | 11 |
|  | 2 | 10-3 | 5 |
|  | 3 | 10-5 | 5 |
|  | 4 | 10-4 | 2 |
|  | 5 | 10-3 | 12 |
|  | 1 | 10-5 | 19 |
|  | 2 | 10-4 | 5 |
|  | 3 | 10-4 | 4 |
|  | 4 | 10-5 | 2 |
|  | 5 | 10-5 | 12 |
|  | 1 | 10-5 | 2 |
|  | 2 | 10-3 | 5 |
|  | 3 | 10-4 | 5 |
|  | 4 | 10-4 | 6 |
|  | 5 | 10-3 | 5 |

**Table 4.4.**RBFNN hyperparameters for different orders of POD coefficients of the y-velocity.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mode order | “l2” | “num\_centers” |
|  | 1 | 10-3 | 4 |
|  | 2 | 10-3 | 9 |
|  | 3 | 10-4 | 14 |
|  | 4 | 10-3 | 4 |
|  | 5 | 10-3 | 2 |
|  | 1 | 10-3 | 3 |
|  | 2 | 10-4 | 9 |
|  | 3 | 10-3 | 3 |
|  | 4 | 10-5 | 2 |
|  | 5 | 10-5 | 13 |
|  | 1 | 10-5 | 2 |
|  | 2 | 10-5 | 15 |
|  | 3 | 10-4 | 5 |
|  | 4 | 10-5 | 19 |
|  | 5 | 10-5 | 16 |

**Table 4.5.**RBFNN hyperparameters for different orders of POD coefficients of the z-velocity.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mode order | “l2” | “num\_centers” |
|  | 1 | 10-5 | 2 |
|  | 2 | 10-3 | 4 |
|  | 3 | 10-5 | 2 |
|  | 4 | 10-4 | 13 |
|  | 5 | 10-4 | 19 |
|  | 1 | 10-4 | 5 |
|  | 2 | 10-5 | 16 |
|  | 3 | 10-3 | 2 |
|  | 4 | 10-3 | 2 |
|  | 5 | 10-5 | 2 |
|  | 1 | 10-5 | 2 |
|  | 2 | 10-3 | 5 |
|  | 3 | 10-3 | 2 |
|  | 4 | 10-4 | 9 |
|  | 5 | 10-4 | 7 |